where

$$p = P/4a_1\tau; \quad \mu_1^* = \mu_1 v_0/\tau h; \quad \mu_2^* = \mu_2 v_0/\tau h\lambda.$$

A solution of the nonlinear system of equations with respect to the values of $\mu_1^*$, $\mu_2^*$, c, h, corresponding to a minimum evaluation of (3.3), can be obtained by successive approximations. We limit ourselves to the first approximation. Determining $\mu_1^*$, $\mu_2^*$, h from a minimum of the evaluation of (3.3) with c = 0, we find

$$p \leqslant 2 + (1+\lambda)/2\sqrt{\lambda}, \quad \mu_1^* = \mu_2^* = 1, \quad h = \sqrt{a_1 a_2}.$$

Determining c from the condition of a minimum of the evaluation of (3.3) with the values found for $\mu_1^*$, $\mu_2^*$, h, we obtain

$$p \leqslant 11/6 + (1+\lambda)/2\sqrt{\lambda}, \quad c = -2\sqrt{\lambda}/(1+\lambda). \tag{3.4}$$

In the case of an ideal rigidly plastic medium, the problem under consideration with $l \geq 0.5$ has solutions (Fig. 3) analogous to the solution of Hill and Prandtl for the impression of a single punch [1]. According to these solutions

$$p = 1 + \pi/2. \tag{3.5}$$

The dashed line in Fig. 4 corresponds to an evaluation of (3.4) and the solid line to the value of p according to (3.5). The difference between the evaluation and the value of p according to (3.5) with $0.5 \leq \lambda \leq 2$ does not exceed 12%. The considerable difference with larger values of $\lambda$ is explained by the fact that, in this case, a continuous plastic zone is not formed in the surface layer of the medium and, consequently, the condition $a_1 + a_2 = l$ corresponds poorly to the actual picture of the deformation. In this case, a better evaluation can be obtained by assuming that, in the region $a_1 + a_2 < x \leq l$, the velocities are equal to zero, and determining $a_2$ from the condition of a minimum of the evaluation.

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# MEASUREMENT OF MULTIPOINT MOMENTS OF COMPOSITE STRUCTURES


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UDC 539.2:531


Problems of thermal conductivity, elasticity, and the viscosity of multiphase solid materials have solutions in the form of expansions with respect to multipoint moments. However, the use of these solutions is limited by the difficulty in determination of the multipoint moments, giving the random field of the parameters, which are the starting point for a theoretical investigation. Out of the existing quantitative methods of statistical experiment [4, 5], the most promising is optical-structural analysis, based on the storage of a signal, obtained from the output of a pickup of the optical density, with scanning of the structure. Subsequent automated introduction of the recording of the signal into an electronic computer would make it possible to use any given algorithms for calculation of the parameters of the structure.

For justification of the experimental method of determining multipoint moments, let us consider the statistically homogeneous random fields $\Lambda$ in the space of the Cartesian coordinates $X_i$. The multipoint moments are determined as a result of averaging

$$\langle \Lambda(X)\Lambda(X')\Lambda(X'')\ldots\rangle = M(X'-X, X''-X, \ldots). \tag{1}$$

For a composite medium, the assignment of the random field of the determining parameters is represented in the form of independent characteristics, relating to the physical properties of each phase separately, and the geometry of the distribution of the phases in space. Let $\Lambda_i$ be some physical parameter, corresponding


Kuibyshev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 145-149, November-December, 1977. Original article submitted November 24, 1976.




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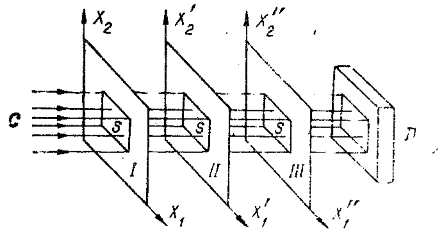


Fig. 1

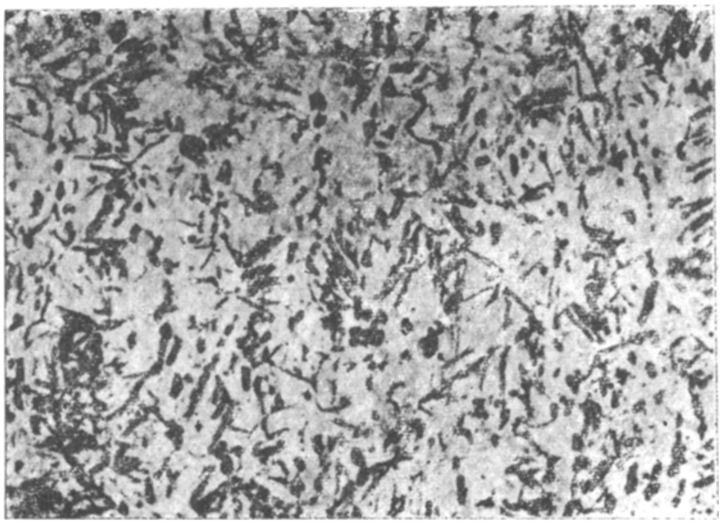

Fig. 2

to the i-th phase, for example, the tensor of the elastic moduli, the density, the dielectric permittivity, etc. The geometry of the phase is given by the function $\varkappa_i$, taking on the value 1 in the region of space occupied by the i-th phase and the value 0 in the remaining region. The random field of the parameter $\Lambda$ for an (N+ 1)-phase medium can be represented in the form

$$\Lambda = \Lambda_0 + \sum_{i=1} (\Lambda_i - \Lambda_0)\varkappa_i(X), \quad \varkappa_i\varkappa_j = \begin{cases}0, & i\neq j,\\ \varkappa_i, & i=j.\end{cases} \tag{2}$$

Substitution of expression (2) into relationship (1) shows that the multipoint moments of the field $\Lambda$ are expressed by a linear combination of mixed multicomponent moments of the geometric structures $\varkappa_i$. Thus, the investigation of the geometrical structure can be made independently of a determination of the physical parameters. The possibility of the complete assignment of the random structure in the form of determined characteristics is contained in the conditions for the existence of the characteristic functional [6]

$$\Phi(\theta) = \left\langle \exp\left[i\sum_{j=1}^{N}\int \varkappa_j(X)\theta_j(X)\,dX\right]\right\rangle. \tag{3}$$

The expansion of the functional (3) in a series with respect to functionals of the n-th order is given by n-point moments. In this sense, the assignment of n-point moments can be adopted as an overall representation of the variety of random structures. The assignment of n-point moments is the most complete and objective criterion of the discrimination of the structure. The universality of the representation of the random structure in the form (3) or by n-point moments consists of the fact that any given statistical characteristics can be expressed in terms of the parameters of the n-point moments.

Let the image of the structure of some phase be represented on a black-transparent photo-plate; here values of $\varkappa(X_1, X_2) = 1$ correspond to the transparent regions of the image. Figure 1 shows the scheme of a unit for investigation of plane structures. A parallel beam of light C from the area of the cross section S passes through the photo plates I, II, III, which have identical images of the structure. The light flux, recorded by a photometer, is proportional to the area of the transparent region in the plates. With complete superposition of the images of the structure on plates I-III, the absorption of light, inhomogeneous over the area, takes

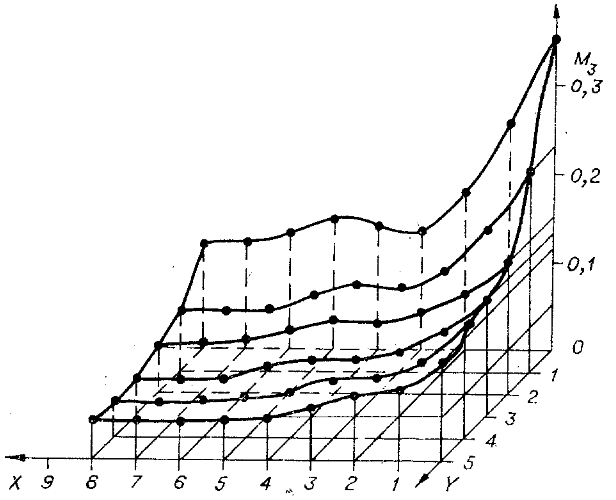


Fig. 3

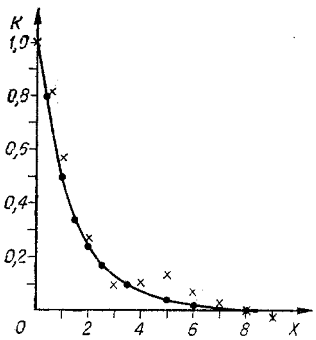


Fig. 4

place only in the plate I. In this case, the area of the transparent region, referred to the area S, coincides with the ratio of the light fluxes for the black-transparent and transparent plates; here the ratio is equal to the concentration of the transparent regions.

Let the plate I be fixed, while plates II, III move in the directions $(X_1', X_2')$ and $(X_1'', X_2'')$. In this case, for all the plates the transparent region corresponds to the region for which the equality

$$\varkappa(X_1, X_2)\varkappa(X_1', X_2')\varkappa(X_1'', X_2'') = 1.$$

is satisfied. The area of this region is equal to

$$S_M = \int \varkappa(X_1, X_2)\varkappa(X_1', X_2')\varkappa(X_1'', X_2'')\,dX_1dX_2.$$

Postulating the ergodicity of the homogeneous structure, the mathematical description can be calculated by averaging over the area.

For a three-point moment, we obtain

$$M_3(X_1' - X_1,\ X_2' - X_2,\ X_1'' - X_1,\ X_2'' - X_2) = S_M S^{-1}$$

The ratio $S_M S^{-1}$, recorded by the photometer as the ratio of the light fluxes with different combinations of the shifts of the plates, measures a function of four variables. For a three-dimensional isotropic structure, $M_3$ is determined by the mutual arrangement of three points. Therefore, the investigation of one flat slide in three photo-plates is sufficient for measurement of $M_3$ as a function of three variables, characterizing the mutual arrangement of three points of any given plane of the space. With the measurement of four-point moments, four photo-plates are used; here not only with identical images, but also with photography of sections in the planes $X_3 = \text{const}$ at different distances, which are small in comparison with the characteristic dimensions of the inhomogeneities of the structure. For measurement of moments of the n-th order, new sections are not required. Depending on the coordinates of the points at which the measurement is being made, a packet of plates is made up for illumination. With investigation of mixed moments of an N-phase medium, photos are made of one slide with the absorption regions $\varkappa_i$, $\varkappa_j$, which can be obtained using special etching methods, making it possible to separate out for photography individual phases from an N-phase set. The analogy between the light flux and the absorption of light, averaged over the area, with a shift of the photo-plates and multiplication of the functions $\varkappa$ at different points, enables an experimental investigation of more general structures than those indicated above.

In an experimental investigation of a structure, the dimensions of the photographic image (Fig. 2) and cross section of the beam, were equal, respectively, to $81\times 111$ mm$^2$ and $S=45\times 46$ mm$^2$. The spacing with a shift of the photo-plates was equal to 0.5 mm. A Φ-107 semiconductor photoelement and an R-307 potentiometer were used. The results of measurement of a three-point moment as a function of $M_2(X, Y)$, where $X=X_1' - X_1$, $Y=X_2'' - X_2$, are shown in Fig. 3.

An investigation of the structure of the distribution of martensite in steel Kh12M [7] was made by numerical methods. After the application of a square grid, values of the function $\varkappa=0$, 1 were introduced into the computer. In Fig. 4 the results of calculations [7] of a normalized two-point correlation function are compared with the experimental data obtained, whose values are marked with crosses. The graduation along the X and Y axes in Figs. 3, 4 is $5\cdot 10^{-3}$ mm and is equal to the mean diameter of a martensite needle.

The structure in question has only an insignificant anisotropy, which is the result of the asymmetrical character of $M_3(X, Y)\neq M_3(Y, X)$. Large-scale inhomogeneity was observed in the experiment; therefore, the results of the measurements were averaged over values obtained with a parallel shift of the source of light (see Fig. 1). The order of the deviation was 2-5%.

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